



## Effects of background anisotropy on effective elastic properties of fractured rocks

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### Introduction

Fractures are widely distributed underground. Hence, seismic detection and characterization of fractures are of great importance in many disciplines, such as oil/gas exploration and production, CO<sub>2</sub> sequestration, nuclear waste storage, among many others. To this end, rock physics models provides the link between fracture properties and seismic attributes. However, in the previous work, most models neglect the background anisotropy (e.g., Hudson, 1980), which may exist in many fractured formations. Hence, in this work, we derived the analytical solutions for elastic properties of rocks with dry or fluid-filled penny-shaped cracks embedded in the isotropic plane of the transversely isotropic (TI) background medium. To validate the results, we compared theoretical predictions to ultrasonic measurements on a synthetic cracked sandstone sample with TI background.

### Methodology

The general solutions for effective elastic properties of rocks containing ellipsoidal inclusions were given by Sevostianov and Kachanov (1999). This was based on Eshelby's solution for the stress field inside the inclusion. Hence, to derive the solutions for the case with penny-shaped cracks embedded in the isotropic plane of TI background medium, we need to obtain the corresponding Eshelby's tensor. This Eshelby's tensor can be derived from that for spheroidal inclusion (Withers, 1989). Then, substituting the derived Eshelby's tensor into the general solutions gives the dry fracture compliance matrix:

$$\mathbf{Z} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_N & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_T & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (1)$$

where:

$$Z_N = \frac{8B_4}{3C_1} \frac{e}{1 - C_{13}^2 / C_1^2}, \quad (2)$$

$$Z_T = \frac{16}{3C_{44}} \frac{e}{B_3 + B_4 - \frac{2C_{44}B_4}{C_1 + C_{13} + 2C_{44}}} \quad (3)$$

Here,  $C_{ij}$  is the stiffness coefficient of the TI background medium (which is symmetric with respect to  $x_3$ -axis).  $B_3$  and  $B_4$  are coefficients depending on background elastic properties.  $e$  is the fracture density. When the cracks are saturated with fluid, the fracture compliances can be obtained from the dry case using Brown - Korrington equation (Brown and Korrington, 1975). Then rock elastic properties can be calculated using linear – slip theory.

## Results

To illustrate the effects of background anisotropy on elastic properties of fractured rocks, we compare theoretical predictions with ultrasonic measurements on a synthetic cracked sandstone sample with TI background. The results are shown in Figure 1. We can see that, the agreement between theoretical predictions and experimental results improves notably after considering the background anisotropy.

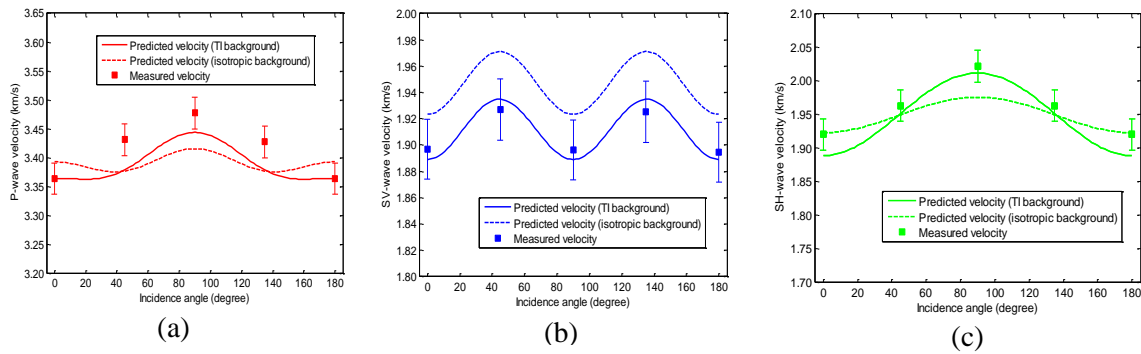


Figure 1. Comparison between theoretical predictions and experimental results. The measurement errors for the P- and S- waves are 0.8% and 1.2% respectively, which are denoted through the error bars.

## Conclusions

In this work, we derived the analytical solution for elastic properties of rocks with dry or fluid-filled penny-shaped cracks embedded in the isotropic plane of TI background medium. Comparison of theoretical predictions with experimental results showed that the background anisotropy can play an important role and hence needs to be considered.

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## References

- Brown, R. J. S., and J. Korrington, 1975, On the dependence of the elastic properties of a porous rock on the compressibility of the pore fluid: *Geophysics*, 40, 608-616.
- Hudson, J. A., 1980, Overall properties of a cracked solid: *Mathematical Proceedings of the Cambridge Philosophical Society*, 88, 371–384.
- Sevostianov, I., and M. Kachanov, 1999, Compliance tensors of ellipsoidal inclusions: *International Journal of Fractures*, 96, L3-L7.
- Withers, P. J., 1989, The determination of the elastic field of an ellipsoidal inclusion in a transversely isotropic medium and its relevance to composite materials: *Philosophical Magazine A*, 59, 759-781.