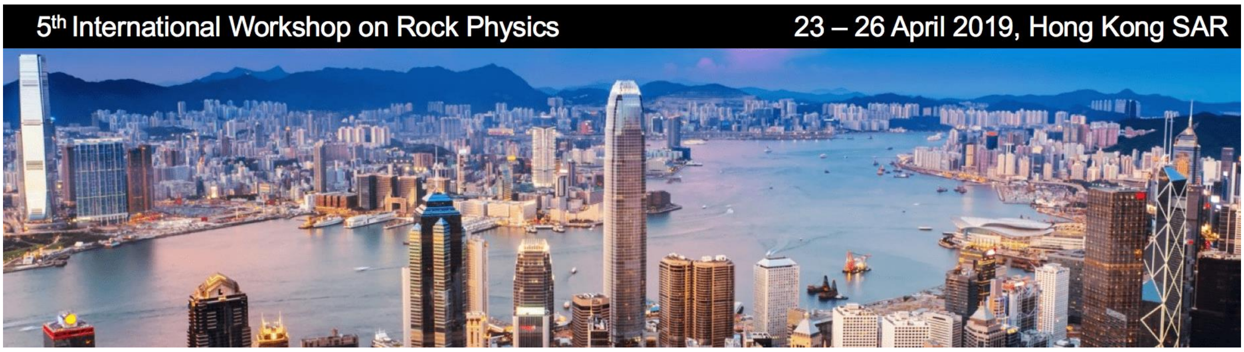


# Drag force theory for permeability

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## Introduction

The porosity and permeability are primary characteristic parameters of a porous medium. The porosity is the ratio of the total pore to total bulk volume. The permeability, introduced via Darcy's law, is proportional to the flux of flow through the pore structure due to an applied pressure gradient. Many experimental investigations show that there is a correlation between porosity and permeability and often the Kozeny-Carman (KC) model or modifications/extensions of the KC model are used to predict the porosity-permeability relation. However, the main critique of the KC-type models is that there is always some empirical constant which can be only estimated by fitting experimental data but cannot be calculated independently. Moreover, in a porous medium sample with non-uniform porosity more than one empirical constant would be required. Hence, the KC model is not able to capture the effect of porosity fluctuations. To overcome this limitation, we devise a novel theoretical approach. The starting point is the volume average of interfacial interaction, or macroscopic drag force, between solid and fluid phase (Sahay et al.,. 2001)

$$\overline{I_j} = -\eta_0^2 \mu^f (\kappa^{-1})_{jk} \left(\bar{v}_k^s - \bar{v}_k^f\right) - \bar{p}^f \partial_j \eta_0 \qquad (1)$$

It is chosen such that there is consistency with Darcy's law. Comparing this drag force with the general drag force representation in the volume averaging framework and making a single solid inclusion approximation we find that the inverse of the permeability tensor $(\kappa_{jk})$ is the sum of the inverse of the permeability pertaining to a representative volume element $(\kappa_{jk}^h)$ and the second variations of porosity $\eta_0$

$$\frac{1}{\kappa_{jk}} = \frac{1}{\kappa_{jk}^h} + \frac{1}{\eta_0^2}\left(\partial_j \partial_k + \partial_l \partial_l \delta_{jk}\right)\eta_0 \qquad (2)$$

Equation (2) provides us with the new insight that the gradient of porosity increases or reduces the permeability. In KC-type models, this possibility has not been considered. We believe that this result is relevant for applications where porosity maps are converted into permeability maps.

## Application: From porosity to permeability maps

In this section, we are interested in evaluating the heterogenous permeability mapping from porosity map. This application test for the porosity-permeability correlation has been presented herein briefly. To do that we use analytical homogenous permeability and using the window averaging (as an upscaling algorithm) to create an ensemble of random sphere porosity and permeability maps.

- **A porosity map**

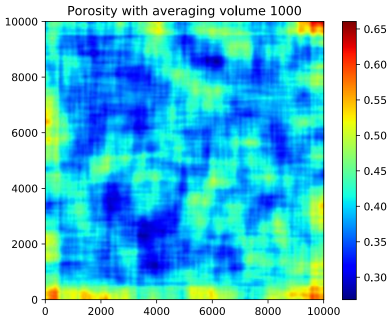
Assume a random sphere pack, where spheres have a radius equal to 50 μ-meter. Then we set the target porosity 0.4, and randomly distribute these spheres. Then we use the window averaging to upscale the porosity map, for example, porosity map, averaged with 1000x1000 window size. These are shown in figure 1(a).

- **A homogenous permeability map**

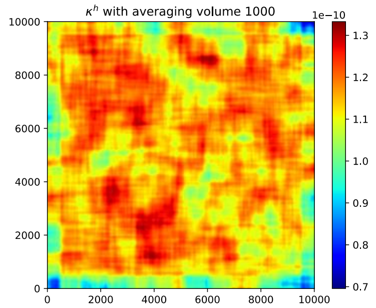
Then by using the analytical permeability formula for the spherical inclusions (Scheidegger 1974), the homogenous permeability map has been created. The upscale map of homogenous permeability is shown in figure 1(b).

- **The heterogeneous permeability map**

Here we use the equation (2) to evaluate the heterogeneous permeability map by taking the porosity variations into account. Figures 1(c) and 1(d) show the estimated diagonal elements of heterogeneous permeability tensor.




(a)

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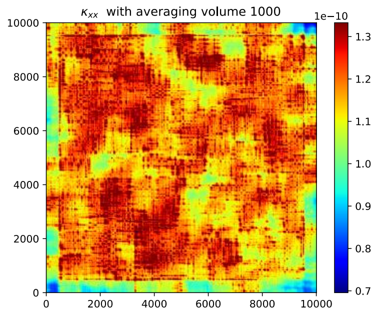
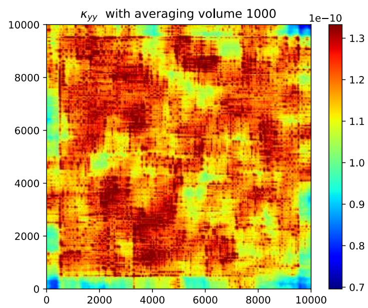



(c)




(d)

*Figure 1: (a) A porosity map, (b) the permeability map (c) and (d) the heterogeneous permeability's elements.*

## Conclusions

Here in this abstract, we present a heterogeneous permeability formula which it will the spatial variations of porosity into account. In our application test, we have shown the heterogeneous permeability elements (figures 1(c) and 1(d)) have more variations locally than homogenous permeability map (figure1(b)). Moreover, one can conclude that the permeability of heterogeneous media is correlated to its local variation of porosity. This has been missing from previous effective media theories of permeability and upscaling algorithm. Also, the equation(2) can predict the anisotropy of permeability based on the anisotropy of porosity map.

### References

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