From critical angles to elastic parameters using shear-wave information

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Introduction

Data obtained by acoustic waves passing through the Earth's subsurface are rich in information. Extraction of this information encoded in seismic data is not easy. Traditionally, PP seismic data are measured and many inversion schemes exist to invert these for the elastic parameters. All of the schemes have one thing in common: they have in-built empirical relations in order to cure the undetermined nature of the problem. The need for empirical relations vanishes when joint PP-PS inversion methods are considered.

Pure-shear surface seismic surveys have also been used in exploration (Stewart et al., 2002). Collection of either SP or SS waves for some interfaces theoretically opens up the possibility to develop a simple and exact inversion scheme by which all four elastic parameters entering Zoeppritz's equations can be obtained. The key to the method is the observation of three critical angles which may be directly linked to elastic properties of the subsurface.

Methodology

The goal of this paper is to present the simplest possible method for the exact inversion of all four elastic parameters that SP and SS Zoeppritz's equations depend on using nothing but special angle observations. In their most compressed form, they take the following form (e.g. for SS-wave):

$$SS(\theta) = SS(AI, \rho, \gamma_1, \gamma_2, \theta) \qquad \text{(Eq. 1)}$$

where

$$AI = \frac{AI_2}{AI_1}, \rho = \frac{\rho_2}{\rho_1}, \gamma_1 = \frac{V_{P_1}}{V_{S_1}}, \gamma_2 = \frac{V_{P_2}}{V_{S_2}} \qquad \text{(Eq. 2)}$$

are acoustic impedance ratio, density ratio and Vp/Vs ratios in layers 1 and 2, respectively.

For the incident SV-wave, the number of critical angles is between 1 and 3. These are (Červený, 2001):

$$\theta_{c_1} = arcsin\frac{V_{S_1}}{V_{P_2}} = arcsin\left(\frac{1}{\gamma_1}\frac{\rho}{AI}\right) \qquad \text{(Eq. 3)}$$

$$\theta_{c_2} = arcsin\frac{V_{S_1}}{V_{P_1}} = arcsin\left(\frac{1}{\gamma_1}\right) \qquad \text{(Eq. 4)}$$

$$\theta_{c_3} = arcsin\frac{V_{S_1}}{V_{S_2}} = arcsin\left(\frac{\gamma_2}{\gamma_1}\frac{\rho}{AI}\right) \qquad \text{(Eq. 5)}$$

From Eq. 3-5 it is a trivial exercise to obtain γ_1, γ_2 and ρ/AI parameters. One more equation is needed in order to invert for the fourth parameter, namely the acoustic impedance ratio. We invert the Eq.1 to obtain the sought parameter:

$$AI = AI\left(\frac{\rho}{AI}, \gamma_1, \gamma_2, \theta_0, SS(\theta_0) = 0\right) \qquad \text{(Eq. 6)}$$

where θ_0 is the angle at which SS-wave has a value of zero. Note that in Eq. 6 we use a slightly different parameterization compared to Eq. 1 so that the input parameter is not ρ, but ρ/AI, obtained from combinations of Eq. 3 and Eq. 4.

The algebraic manipulation of Eq. 1 to express AI in the form shown in Eq. 6 leads to a simple quadratic equation with two solutions, one of which is always a negative, thus non-physical. The physically acceptable solution takes the following form:

$$AI = \frac{-c_2 + \sqrt{c_2^2 - 4c_1c_3}}{2c_1} \qquad \text{(Eq. 7)}$$

where c_1, c_2 and c_3 coefficients are too complicated to be presented here.

Results

We test the method on a medium parameters typically encountered in shallow structure seismic exploration. The emerging RC-s are shown in Figure 1. The results are summarized in Table 1. Clearly, in noise-free case the method returns the exact parameters.

Parameter	True	Inv SP	Inv SS
AI	1.384	1.384	1.384
ρ	1.107	1.107	1.107
γ_1	1.732	1.732	1.732
γ_2	1.732	1.732	1.732

Table 1: True and inverted parameters.

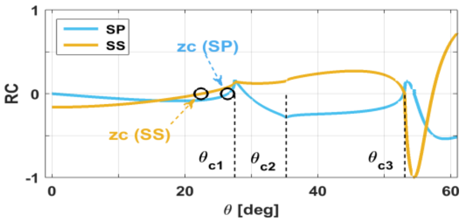

Figure 1: SP and SS RC-s in function of the incident angle. Zero-crossings (zc) that are used in inversion of AI ratio are marked by black circles. Moreover, the locations of 3 critical angles are emphasized by black dashed lines.

Conclusions

We introduced a simple inversion scheme to obtain all four elastic parameters entering Zoeppritz's equations solely from observation of four special angles: three critical angles and an angle where SP (SS) RC-s have a value of zero. Assumptions that must be met for the method to work and calculations to be valid:

1. SP or/and SS RC-s are available
2. Assumptions valid in deduction of Zoeppritz's equations hold true (elastic plane waves at a non-slip horizontal boundary between two semi-infinite isotropic elastic media)
3. All three critical angles for SP or/and SS waves exist and can be observed

References

Červený, V. (2001)., Seismic Ray Theory (pp. 491-492). Cambridge: Cambridge University Press.

Stewart, R.R., Gaiser, J.E., Brown, J. and Lawton, D. C. (2002), Converted-wave seismic exploration: Methods, Geophysics, Vol. 67, P. 1348-1363.