How fast fluid pressure diffuses in a deformable porous rock?

Tobias Müller and Pratap Sahay

Department of Seismology, CICESE, Mexico

Contact email: tobias@cicese.mx

Introduction

Fluid pressure diffusion processes are of considerable interest in rock characterization using geophysical monitoring techniques. For example, the injection of fluid in a borehole can generate detectable microseismic events triggered by fluid pressure perturbations. These pressure perturbations are equilibrated through a diffusion process. The microseismic events, in turn, are used to infer dominant fluid pathways in a formation of interest and to estimate the large-scale permeability from their spatio-temporal evolution. Another example, is the analysis of conversion from propagating seismic waves into diffusive (slow) waves. This results in attenuation of seismic waves which can provide information on the size and distribution of sub-wavelength heterogeneities.

To make such monitoring techniques work, it is essential to understand the nature of the diffusion process. For that the underpinning physics at the pore-scale level becomes important. Particularly, how the fluid phase interacts with the deformable solid matrix; whether this interaction fluid proceeds in a compressible or incompressible manner. We use the method of volume averaging to analyze diffusion processes in deformable porous rocks at macroscale and derive an expression for the diffusion constant that encompasses the compressible and incompressible flow limit.

Methodology

At pore-scale the solid and fluid phase interact through the deformation of the pore interface. There are several possibilities how this interaction proceeds. In the situation that the pore interface is not deformable, the solid phase is not taking part and the resulting diffusion process is expected to be independent of the elasticity of the solid. Whenever the pore-interface is deformable the elasticity of the solid matters. In this case the solid and fluid macroscopic pressures interact in a reciprocal or non-reciprocal manner mediated through porosity change. What is more, in the limit of an incompressible flow the elasticity of the fluid becomes irrelevant.

In the volume averaging poroelasticity framework the sum of the displacements of the pore-interface in direction of the surface normal within the averaging volume is interpreted as the change of porosity $\eta - \eta_0$, where η_0 is the porosity of the unperturbed (undeformed) rock. To capture the above-mentioned interactions at macroscale the porosity perturbation equation is taken as a function of the macroscopic solid (p_s) and fluid pressure (p_f),

$$\eta - \eta_0 = -(1-\eta_0)\,\frac{\alpha-\eta_0}{K_0}\,\left(p_s - n\,p_f\right), \tag{1}$$

where K_0 is the bulk modulus of the drained rock frame. This means that the pore-interface deformation is characterized at macroscale by two parameters. The lumped parameter $\alpha - \eta_0$ with the Biot coefficient α quantifies the pore-interface deformability and the parameter n makes the interaction reciprocal (n=1) or non-reciprocal ($0 < n < 1$). Using this porosity perturbation equation, we obtain two of pressure equations which together with the equations of motion for the solid and fluid phase in the quasi-static limit from a complete set of equations for the differential pressure and the increment of fluid content.

Results

Based on the procedure outlined a diffusion equation for the increment of fluid content and fluid pressure is obtained with the diffusion coefficient (Müller and Sahay, 2018). It reads

$$D = \frac{\kappa}{\mu_f}\left(\frac{\eta_0}{K_f} + \frac{n(\alpha-\eta_0)}{K_s} + \frac{\alpha\left(\eta_0+n(\alpha-\eta_0)\right)}{K_0+\frac{4}{3}\mu_0}\right)^{-1}, \tag{2}$$

where κ= permeability, μ_f= fluid shear viscosity, K_f= fluid bulk modulus, K_s= solid bulk modulus and μ_0=shear modulus of the rock frame. Eq. (2) is the main result. From Eq. (2) several special cases can be deduced. If *n=1* then the interaction is reciprocal in the solid and fluid pressures (see Eq. 1) and the Biot diffusion constant is recovered

$$D^{(comp)} = \frac{\kappa}{\mu_f}\left(\frac{\eta_0}{K_f} + \frac{(\alpha-\eta_0)}{K_s} + \frac{\alpha^2}{K_0+\frac{4}{3}\mu_0}\right)^{-1} \quad . \tag{3}$$

In practical applications the 2nd and 3rd terms are often neglected so that the approximation

$$D^{(comp)} \sim \frac{\kappa}{\mu_f}\quad\frac{K_f}{\eta_0} \tag{4}$$

is obtained. For typical rock parameters the difference between the diffusion constants (3) and (4) is indeed small. Therefore, the reciprocal interaction in the Biot theory results in a diffusion constant for which the elasticity of the fluid dominates while the elasticity of the rock frame is absent. In contrast, if incompressible flow with uni-directional interaction (*n=0*) is considered then we find

$$D^{(incomp)} = \frac{\kappa}{\mu_f}\frac{K_0+\frac{4}{3}\mu_0}{\alpha\eta_0} \quad . \tag{5}$$

In this limit it is the elasticity of the rock frame that matters while the fluid elasticity is irrelevant. It turns out that this is the largest possible diffusion constant. This limiting diffusion constant was also obtained in earlier works (e.g., Udey, 2012) using a different route.

Discussion and Conclusions

We observe that in deformable porous rocks the pressure diffusion process is governed by the directionality of the solid-fluid interaction as well. The difference between the diffusion constants for reciprocal and uni-directional interactions can be large (for example, for water-saturated, low-porosity rocks the ratio of $D^{(incomp)}$ and $D^{(comp)}$ can exceed two orders of magnitude. Therefore, if one attempts to estimate the permeability on the basis of the diffusion constant without considering the nature of the solid-fluid interaction then the incurring error will be substantial. We anticipate that these results have implications for the interpretation of microseismic events triggered by fluid pressure perturbations.

Acknowledgements

This work is supported by CICESE (Project 641174) and CONACyT under Grant CB2016-01-286476.

References

Müller, T.M., Sahay, P.N., 2018. Fluid pressure diffusion in deformable porous media: SEG International Exposition and 88th Annual Meeting, 3522-3526.

Udey, N., 2012. Coupled porosity and saturation waves in porous media, in M. A. Diaz Viera, P. N. Sahay, M. Coronado, and A. Ortiz, eds., Mathematical and numerical modeling in porous media: Applications in geosciences: CRC Press, 1–28.