A rock physics model relating static and dynamic anisotropy in shale

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Introduction

Shale, as a typical sedimentary rock, behaves strongly anisotropic. Holt et al. (2015) presented a series of lab tests, showing that the anisotropy of the dynamic modulus from P-wave velocity, the cyclic Young's modulus and the initial Young's modulus by static loading are different, but still related. In this study, we ascribe the rock behavior to lamination and four different sets of cracks. By controlling the constitutive parameters of the lamination and the cracks, this rock physics model can match the measured moduli well.

Methodology

We introduce the impact of thin-flat cracks into a host material, where a modified Backus model with a lamination factor u characterizes the horizontal lamination of the host material, and four crack sets, a b c and d with crack density ζ and drainage constant D (Budiansky and O'Connell, 1976), affect this laminated material. Crack sets a and b represent open and saturated cracks, and crack sets c and d represent closed cracks that may slide during initial static loading (The drainage constants $D_c = D_d = 0$). Cracks a and c are randomly oriented, while b and d are parallel to the lamination.

The stiffness tensor of the laminated host material $\mathbf{C}^l$ is affected by a set of cracks through the crack density ζ and the impact tensor $\mathbf{Q}$. By assuming no mechanical interaction between these crack sets, the effective stiffness tensor $\mathbf{C}^*$ can be expressed as (Fjær et al., 2008)

$$\mathbf{C}^* = \mathbf{C}^l \circ \left(1 - \sum_{i=a}^{d} \zeta^{(i)} \mathbf{Q}^{(i)}\right) \tag{1}$$

where $\circ$ is Hadamard product. The stiffness tensor of the laminated host material $\mathbf{C}^l$ is derived from the stiffness tensor of the isotropic host material $\mathbf{C}^0$ by introducing a lamination factor u, defined as

$$C_{11}^l = C_{22}^l = C_{33}^l \left(1 + 0.9u\right) \qquad C_{44}^l = C_{55}^l = C_{66}^l / \left(1 + u\right) \tag{2}$$

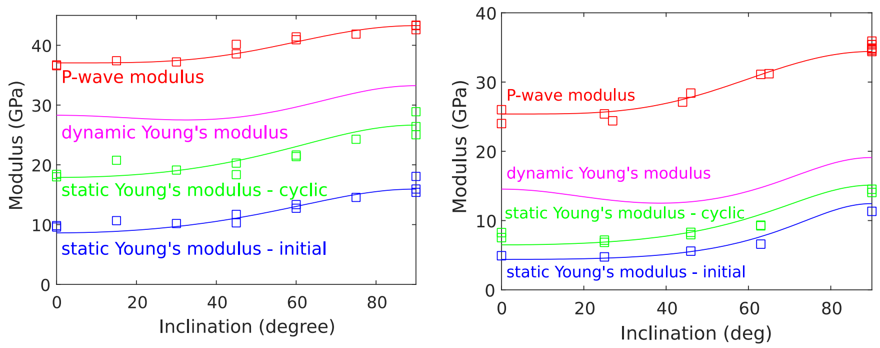
where 0.9 is calculated from the elastic properties of the Field shale tested in the lab (0.6 for Mancos shale). The impact of randomly oriented cracks, $\mathbf{Q}^{(a)}$ and $\mathbf{Q}^{(c)}$, are expressed based on the work by Walsh (1965), Garbin and Knopoff (1975, 1973). The impact of the crack parallel to the horizontal lamination, $\mathbf{Q}^{(b)}$ and $\mathbf{Q}^{(d)}$, are calculated based on the work by Hudson (1981).

Rotating the effective stiffness tensor $\mathbf{C}^*$ along any axis within the horizontal plane yields $\mathbf{C}_\theta^* = \mathbf{R}_\theta \mathbf{C}^* \mathbf{R}_\theta^T$. Inversing $\mathbf{C}_\theta^*$ yields the corresponding compliance matrix $\mathbf{S}_\theta^*$ and the vertical Young's modulus E_v is calculated as the inverse of $\mathbf{S}_{\theta,33}^*$.

We determine the lamination factor u, the crack density ζ_a ζ_b ζ_c and ζ_d, and the drainage constant D_a and D_b as follows: 1. Determine u by matching the measured P-wave modulus and the simulated P-wave modulus $\mathbf{C}^*_{\theta,33}$ only based on the laminated stiffness tensor $\mathbf{C}^l$. It is assumed that cracks c and d are inactive ($\zeta_c = \zeta_d = 0$) due to the low strain amplitude, and that $D_a = D_b \ll 1$ (undrained cracks) due to the high frequency. 2. Determine ζ_a and ζ_b by matching the simulated cyclic Young's modulus E_v^c with the measured one. We assume that $D_a = D_b = 1$ (drained cracks) due to the low strain rate, and that cracks c and d are inactive ($\zeta_c = \zeta_d = 0$). 3. Determine ζ_c and ζ_d by matching the simulated initial Young's modulus E_v^i with the measured one. Here we assume that cracks c and d are active while cracks a and b are drained. Finally, the dynamic Young's modulus E_v^d can be evaluated by assuming $D_a = D_b \ll 1$ (undrained cracks) and $\zeta_c = \zeta_d = 0$ (non-sliding cracks).

Results

By fitting the free parameters, we can match the simulated moduli with the measured ones, as shown in Figure 1. The fitted parameter values (also shown in Figure 1) reveal the physical origin of the dominating causes for both the static and the dynamic anisotropy.



	Mancos	Field
u	0.185	0.600
ζ_a	0.155	0.100
ζ_b	0.015	0.000
ζ_c	0.400	0.330
ζ_d	0.010	0.050

Figure 1: Moduli with respect to inclination for Mancos shale (left) and a Field shale from the Norwegian Continental Shelf (right). Square – measured value; Solid line – simulation.

Conclusions

This simple rock physics model, accounting for lamination and cracks with various orientations, enables identification of the most important origins of static and dynamic anisotropy for a given rock. This may open up for prediction of stress dependent anisotropy.

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