Role of effective pressure coefficient for porosity in poroelastic quasi-static compressibility measurements

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Introduction

Possible errors in experimental measurements of pore-fluid volume variation have been commonly used as a main explanation for the discrepancy between directly measured Biot coefficient α and its indirect estimations; the latter involve combining at least two independently measured poroelastic parameters. As a result, very little attention has been put into examining whether these discrepancies are logical consequences of the way α was theoretically defined. This latter aspect is addressed in this paper by using experimental observations from Pimienta et al. (2017). The authors performed direct measurements of α (which is believed to be the effective pressure coefficient for bulk volumetric strain) on a Bentheim sandstone sample but also used three different theoretical relations to independently infer it. They labelled the three inferred values α_1, α_2 and α_3, respectively:

$$\alpha = \left(\frac{\Delta V_p}{\Delta V_b}\right)_{P_f}, \qquad \alpha_1 = \eta_0 \frac{C_{pc}}{C_{bc}}, \qquad \alpha_2 = \frac{C_{bp}}{C_{bc}}, \qquad \alpha_3 = 1 - \frac{C_s'}{C_{bc}}, \tag{1}$$

where V_p is the pore volume, V_b is the bulk volume, C_{bc}, C_{pc} and C_{bp} are bulk, pore-space and pseudo-bulk compressibilities, respectively in Zimmerman (1991) notations, and C_s' is the unjacketed bulk compressibility of the solid matrix[1,2]. According to the considered theoretical frameworks (Biot and Willis, 1957, Geertsma, 1957; Zimmerman, 1991), all the four theoretical relations yielding α, α_1, α_2 and α_3 have to be the same. It is, however, not the case (Fig. 1a)[3]. Yet, following the effective pressure equation for porosity defined by Sahay (2013) (we will label it here as Sahay model) and developed through a series of studies over the recent years (e.g., Müller and Sahay, 2016), the Biot coefficient α and the effective pressure coefficient for bulk volume (labelled as α* within the Sahay model) is one and the same quantity only for the limiting case of homogeneous deformation. The latter (as pointed out by Geertsma, 1957) happens when changes in unit bulk volume, unit pore volume and unit solid volume are the same. In case of inhomogeneous deformation, porosity change $(\eta - \eta_0)$ is proportional to the effective pressure with coefficient for porosity $n \neq 1$ (see eqn. 2) and α and α^* are distinct (see eqn. 3) (Sahay, 2013):

$$\eta - \eta_0 = -(1-\eta_0)\delta_{K_s}\, C_{bc}\,(p_c - (\eta_0 + (1-\eta_0)n)\, p_f), \tag{2}$$

$$\alpha = \eta_0 + (1-\eta_0)\delta_{K_s} = 1 - \frac{C_s}{C_{bc}} \quad \neq \quad \alpha^* = \eta_0 + (1-\eta_0)\delta_{K_s} n = 1 - \frac{C_s'}{C_{bc}}, \tag{3}$$

[1] There is a typo in the Pimienta et al. (2017)'s theoretical definition of α (see their Table 1, line 2). A minus sign is mistakenly introduced before the formula.

[2] It is to be noted that for the unjacketed bulk compressibility of the solid matrix, Pimienta et al. (2017) uses the symbol C_s.

[3] There is a typo in the α_1, α_2 and α_3 data plotted in Pimienta et al. (2017, see their figures 10c and d). The α_1, α_2 and α_3 data we are reporting here in figure 1a were recalculated based on the definitions of each of these parameters (see eqn. 1). To ensure the accuracy of our results, we had a communication with the corresponding author and we were able to confirm the accuracy of C_{bc}, C_{pc} and C_{bp} data used in our calculation.

where C_s is the bulk compressibility of the grain forming the solid matrix, $\delta_{K_s}(= 1 - C_s/((1-\eta_0)C_{bc}))$ is the measure of the decrement of the solid-phase bulk modulus. p_c and p_f are the confining and pore-fluid pressures, respectively. If n = 1 in Eqn. 2, the porosity perturbation equation in the Biot/Geertsma/Zimmerman model is obtained, which is consistent with the homogeneous deformation regime. In this paper, the Sahay model is tested against Pimienta et al. (2017)'s data to assess if the model predicts the different trends displayed by α, α_1, α_2 and α_3 (Fig. 1a).

Methodology and results

The equality $\alpha = \alpha_1 = \alpha_2 = \alpha_3$ means geometrically similar deformation, and this must result in n = 1. Conversely, for an inhomogeneous deformation case, it occurs that $\alpha = \alpha_1 \neq \alpha_2 = \alpha_3$, then $n \neq 1$.
We obtained $n \neq 1$ (Fig 1b), which implies that the tested sample did not deform in an homogeneous way. Hill average equation (Hill, 1952) was used to obtain C_s, which was then used to solve α (see left part of eqn. 3) and α_1. This allowed to discriminate experimental errors reported in the direct measurements of α and C_{pc} by Pimienta et al. (2017). α_2 was inferred through the Sahay model to account for possible errors in the direct measurements of C_{bp}. In figure 1c, α_3 and the corrected α, α_1 and α_2 data are plotted and, as the calculated n values predict, $\alpha = \alpha_1 \neq \alpha_2 = \alpha_3$.

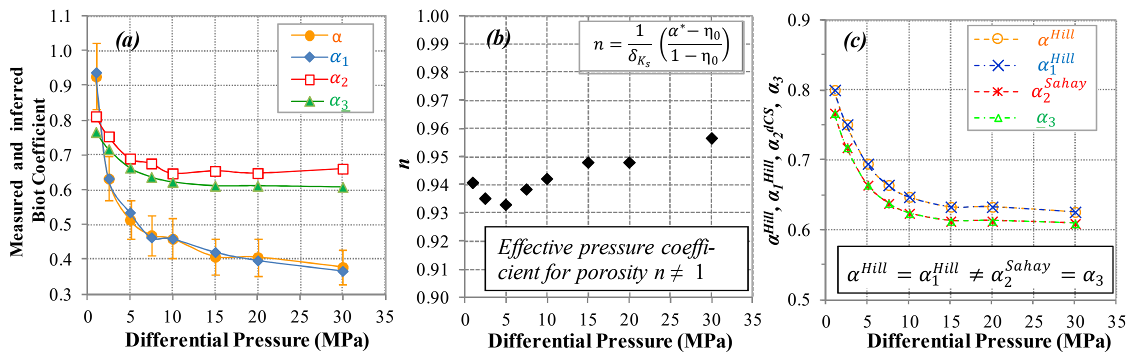

Figure 1: (a) The four representations of Biot coefficient in Pimienta et al. (2017). (b) The coefficient n. It is to be noted that $\alpha^* = \alpha_3$; see also far-right parts of eqns. 1 and 3. (c) α_3 and the corrected α, α_1 and α_2 data. The superscripts indicate the models used to derive the referred parameters.

Discussion and Conclusion

The effective pressure coefficient for porosity as defined by the Sahay model is able to account for the disparity displayed by α, α_1, α_2 and α_3 in Pimienta et al. (2017)'s experimental works. The latter belong to a series of studies where attempts have been made to directly measure the Biot coefficient α and also to indirectly infer it (yielding to the so-called α*). The minor discrepancy usually observed between α ($\equiv \alpha_1$ in our study) and α^* ($\equiv \alpha_2$ and α_3 in our study) is often ascribed to experimental errors rather than to inconsistency with the underpinning Biot/Geertsma theory. Yet, the latter assumes homogeneous deformation and, as our study shows, this is generally not to be expected for natural rocks, whether they appear macroscopically homogeneous (as the sample studied in this paper) or not.

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