Determination of formation shear attenuation from dipole sonic log data

Qiaomu Qi¹, Arthur C. H. Cheng², and Yunyue Elita Li²

ABSTRACT

Formation S-wave attenuation, when combined with compressional attenuation, serves as a potential hydrocarbon indicator for seismic reservoir characterization. Sonic flexural wave measurements provide a direct means for obtaining the in situ S-wave attenuation at log scale. The key characteristic of the flexural wave is that it propagates at the formation shear slowness and experiences shear attenuation at low frequency. However, in a fast formation, the dipole log consists of refracted P- and S-waves in addition to the flexural wave. The refracted P-wave arrives early and can be removed from the dipole waveforms through time windowing. However, the refracted S-wave, which is often embedded in the flexural wave packet, is difficult to separate from the dipole waveforms. The additional energy loss associated with the refracted S-wave results in the estimated dipole attenuation being higher than the shear attenuation at low frequency. To address this issue, we have developed a new method for accurately determining the formation shear attenuation from the dipole sonic log data. The method uses a multifrequency inversion of the frequency-dependent flexural wave attenuation based on energy partitioning. We first developed our method using synthetic data. Application to field data results in a shear attenuation log that is consistent with lithologic interpretation of other available logs.

INTRODUCTION

Attenuation estimated from full-waveform sonic logs provides in situ information about the formation rock and fluid properties (Cheng et al., 1982; Sun et al., 2000). It has many important applications, such as evaluating reservoir gas potential (Klimentos, 1995; Qi et al., 2017), aiding seismic inversion to provide realistic earth materials, and calibrating the rock-physics model for reservoir characterization (Guerin and Goldberg, 2002; Mavko et al., 2005). Continuing efforts are made to develop robust methods for estimating compressional and shear attenuation from full-waveform data.

The formation P-wave attenuation is usually estimated from monopole waveforms by the spectral ratio or centroid frequency-shift methods applied to the earliest cycles of the P-wave arrivals (Cheng, 1989; Sams and Goldberg, 1990; Quan and Harris, 1997). With an appropriate correction for geometric spreading, the monopole compressional attenuation estimate is often close to its actual value (Sun et al., 2000). However, the estimation of the formation S-wave attenuation is significantly more challenging. In a fast formation in which the S-wave velocity is higher than the borehole fluid velocity, the refracted S-wave is followed by the high-amplitude pseudo-Rayleigh wave arrivals. In this case, the formation shear attenuation may be derived from the pseudo-Rayleigh wave based on the energy partitioning (Cheng et al., 1982; Burns and Cheng, 1987). In a slow formation in which the formation S-wave velocity is slower than the borehole fluid velocity, neither the refracted S-wave nor the pseudo-Rayleigh waves are generated in the monopole waveform. Thus, determining the S-wave attenuation from monopole data becomes infeasible.

A potentially more reliable way for measuring S-wave attenuation is to use dipole log measurements. A dipole source excites flexural waves along with refracted P- and S-waves (Kurkjian and Chang, 1986). The flexural wave is a borehole-guided wave that is dispersive (Schmitt, 1988). Due to the geometry of the borehole, its slowness and attenuation can exhibit strong frequency dependence. A key characteristic of the flexural wave is that it propagates at the formation shear slowness and experiences the shear attenuation at low frequencies (Schmitt, 1988). Based on this property, techniques have been developed for identifying the shear slowness from the low-frequency asymptote (Huang and Yin, 2005; Tang et al., 2010; Mukhopadhyay et al., 2013). The natural thought...
is that we may apply the same logic to locate the corresponding shear attenuation from the frequency-dependent flexural wave attenuation at low frequencies. Figure 1b shows a comparison between the theoretical flexural wave attenuation (solid line) and the estimated attenuation (circles) based on the flexural mode spectra given in Figure 1a. The theoretical attenuation is calculated based on the pole-tracking method (Schmitt, 1988). We can see that the estimate matches well with the theory, and indeed the flexural wave attenuation attains the formation shear attenuation at low frequencies.

In addition to the flexural wave, the dipole source will also produce the refracted P- and S-waves in a fast formation. The refracted P-wave, as early arrivals, can be eliminated from the dipole waveforms through appropriate time windowing. However, the refracted S-wave is often embedded in the flexural wave and is difficult to remove. In a fast formation, the refracted S-wave does not interfere with the estimation of S-wave velocity using the dipole waveform dispersion. However, the additional energy loss associated with the refracted S-wave adds to the attenuation estimates and is difficult to separate from the flexural attenuation. Figure 2c and 2f shows the comparison between the theoretical flexural wave attenuation and the estimated attenuation from the dipole waveforms for a slow and a fast formation, respectively. In the slow formation, due to the absence of the refracted S-wave, there exists a good agreement between the theory and the estimate over the entire frequency range. In the fast formation, however, the estimate deviates substantially from the theoretical flexural wave attenuation at low frequencies, although they match each other quite well at the higher frequency band. The discrepancy is caused by the additional amplitude loss due to the propagation of the refracted S-wave. This indicates that direct identification of the formation shear attenuation from the low-frequency portion of dipole waveforms can produce erroneous results, especially when fast formations are encountered.

In this work, we introduce a robust method for calculating the formation shear attenuation from the dipole sonic log data. We first present the procedure for carrying out a frequency-dependent

Figure 1. Synthetic example: (a) amplitude spectra corresponding to the pure flexural wave mode. A shear quality factor of 30 and a mud quality factor of 300 are applied for modeling the amplitude spectra decay. (b) Comparison between the theoretical flexural wave attenuation (solid line) and the estimated attenuation (circles) from the pure mode amplitude spectra in the left panel.

Figure 2. Comparison between theory and estimate: The synthetic array dipole waveforms correspond to (a) a slow and (d) a fast formation. A shear quality factor of 30 and a mud quality factor of 300 are applied for modeling the attenuation effects. Other formation and fluid properties used for modeling the array waveforms are given in Table 1. (b and e) Comparison between the theoretical flexural dispersion (solid line) and the estimated slowness (squares) from the waveforms for a slow and a fast formation, respectively. (c and f) Comparison between the theoretical flexural attenuation (solid line) and the estimated attenuation (squares) from the waveforms for a slow and a fast formation, respectively. The mismatch of attenuation at low frequency in (f) is due to the additional energy loss associated with the refracted S-wave.
attenuation analysis for the dispersive waves. We then propose a new method, which uses a multifrequency inversion of the flexural wave attenuation to obtain the formation shear attenuation. We apply the proposed method to the synthetic and the field dipole sonic log data to demonstrate its applicability.

ESTIMATING FREQUENCY-DEPENDENT ATTENUATION FROM DISPERSIVE WAVEFORMS

Borehole guided waves, such as the flexural wave generated by a dipole source, the Stoneley wave, and the pseudo-Rayleigh waves generated by a monopole source, are typical dispersive wave modes. Consequently, their attenuation and slownesses are frequency-dependent. We can express the amplitude spectrum of the guided wave recorded by the \( i \)th receiver on a sonic logging tool as

\[
X_i(\omega) = A(\omega) e^{-\alpha(\omega)d_i},
\]

where \( \omega \) is the angular frequency and \( \alpha(\omega) \) is the frequency-dependent attenuation coefficient; \( d_i \) is the offset between the source and the \( i \)th receiver; and the function \( A(\omega) \) incorporates the source, the receiver, and the coupling functions. There is no geometric spreading function because the borehole guided wave is a surface wave. Taking the natural logarithm on both sides of equation 1 and rearranging the terms, we have

\[
\ln X_i(\omega) = -\alpha(\omega)d_i + \ln A(\omega).
\]

Equation 2 shows that the log-amplitude spectra is inversely proportional to the offset and the proportional constant is the attenuation coefficient. Thus, by fitting a straight line of the log amplitude spectra as a function of the offset, we can determine the attenuation coefficient by finding the corresponding slope. Repeating the fitting procedure at each frequency, we can calculate the frequency-dependent attenuation coefficient. A more common measure of the attenuation is the inverse quality factor, which can be expressed as

\[
\frac{1}{Q(\omega)} = \frac{2}{\omega} \frac{\alpha(\omega)}{\rho(\omega)},
\]

where \( \rho(\omega) \) is the frequency-dependent slowness. In this work, we use the forward and backward amplitude and phase-estimation method (Li et al., 2015) to calculate the slowness dispersion of the dispersive wave mode independently from the dipole waveforms. It should be noted that conventional \( Q \)-extraction methods, such as the spectral ratio and the centroid frequency shift, assume that the attenuation is constant and is independent of the frequency. Therefore, they are not applicable for analysis of the frequency-dependent attenuation of the borehole-guided wave.

The slowness curves estimated from the synthetic dipole waveforms are plotted as red squares in Figure 2b and 2e for the cases of the slow and the fast formations, respectively. Both plots show that the estimated slowness curves are in good agreement with the theoretical flexural dispersion (the blue line). The slowness approaches the formation shear slowness as the frequency decreases in both cases, which emphasizes the character of the dipole-flexural wave. The frequency-dependent attenuation estimated from the waveforms is plotted as red squares in Figure 2c and 2f. A significant mismatch between the attenuation estimate and the theoretical flexural wave attenuation (or the formation shear attenuation) occurs at low frequencies in the fast formation case. The comparison between the theoretical and the estimated slowness/attenuation curves points out the intriguing fact: The refracted S-wave, as it propagates at the shear slowness which is the same as the flexural wave at low frequency, will not affect the convergence of the dipole dispersion to the formation shear slowness. However, the refracted S-wave experiences amplitude loss and this adds to the observed dipole attenuation. The latter renders the determination of the shear attenuation using the dipole sonic log a challenging task.

DETERMINING FORMATION SHEAR ATTENUATION: A MULTIFREQUENCY INVERSION APPROACH

The dipole source generates the refracted S-wave in addition to the flexural wave in a fast formation. As discussed earlier, the additional amplitude loss due to the propagation of the refracted S-wave leads to a significant discrepancy between the attenuation estimate and the flexural wave attenuation (or the formation shear attenuation) at low frequencies. Thus, one cannot directly identify the formation shear attenuation from the low-frequency asymptote of the dipole attenuation estimate. One key observation from the results in Figure 2f is that despite the discrepancy between the dipole attenuation estimate and the theoretical flexural wave attenuation at low frequencies, the two match fairly well at the frequency range greater than 4 kHz. This is because at higher frequencies, the flexural wave propagates at a velocity slower than the formation S-wave velocity and is thus not contaminated by the refracted shear wave arrival. Above the Airy phase, the particle motion of the flexural wave becomes elliptical and the flexural wave attenuation closely represents the Scholte/Stoneley wave attenuation (Schmitt, 1988). This further separates the flexural wave from the refracted S-wave. In light of this, we can invert for the formation shear attenuation by minimizing the difference between the modeled flexural wave attenuation and the waveform attenuation estimate within the frequency range, i.e., greater than 4 kHz.

Here, we introduce an efficient method for modeling the flexural wave attenuation, which is based on the concept of the energy partitioning for a surface wave. According to Aki and Richards (2002), the spatial attenuation of a borehole guided wave can be expressed as

\[
Q^{-1}(\omega) = \sum_i C_i(\omega)Q_i^{-1},
\]

where \( p, s, f \) stand for the formation P- and S-wave, and P-wave of the borehole fluid, respectively, and \( U \) and \( v \) are the group and the phase velocities of the guided wave. The sensitivity coefficient \( S_i(\omega) \) associated with the flexural wave attenuation can be calculated by the method of Cheng et al. (1982). An example of the sensitivity coefficient in a fast formation (see Table 1) is plotted in Figure 3. We can see that the flexural wave attenuation is mainly sensitive to the shear attenuation at the low-frequency range, whereas at the frequency beyond 4.5 kHz, the mud attenuation dominates.
Recasting equation 4 in a matrix form, we have

\[
\begin{bmatrix}
C_p(\omega_1) & C_s(\omega_1) & C_f(\omega_1) \\
\vdots & \vdots & \vdots \\
C_p(\omega_N) & C_s(\omega_N) & C_f(\omega_N)
\end{bmatrix}
\begin{bmatrix}
Q_{p1}^{-1} \\
\vdots \\
Q_{f1}^{-1}
\end{bmatrix}
= \begin{bmatrix}
Q_{dipole}(\omega_1) \\
\vdots \\
Q_{dipole}(\omega_N)
\end{bmatrix}.
\] (6)

The linear operator of the system consists of the sensitivity coefficients (scaled by the group to phase velocity ratio) at \( N \) different frequencies, which can be calculated by equation 5. The data vector is the frequency-dependent attenuation, which is estimated from the dipole waveforms. We can now formulate a linear optimization problem, i.e., \( \min \|Ax - b\|_2 \) for solving the compressional, the shear, and the mud attenuation. Here, the linear least-squares problem is solved by a multidimensional grid search method, which allows us to gain insights about the uncertainties of the inverted parameters. The frequency window for carrying out the inversion is set to be between 4 and 7 kHz. The percentage error of the inversion results is shown in Figure 4. Compared with the mud and the formation compressional attenuation, the formation shear attenuation is best constrained in the inversion. The resulting shear quality factor is \( Q_s = 29 \), which is within 3.5% of the model defined value \( Q_s = 30 \). We also test the procedure by running the inversion at different frequency ranges, i.e., 5–7 and 6–7 kHz and the resulting \( Q_s \) are 29 and 28, respectively. This suggests that the choice of the length and the position of the window for inversion is relatively flexible. We note that direct identification of the shear attenuation from the estimate at low frequencies (see Figure 2f) would provide an S-wave quality factor of 15, which is half of the model defined value. In comparison, the inversion procedure significantly improves the accuracy of the result. In Figure 5, the diagram summarizes the proposed workflow for computing the dipole S-wave attenuation.

### FIELD-DATA APPLICATION

In this section, we apply the workflow on a field data set to test its applicability. The chosen section resides in a shallow region and has a thickness of 150 m. The formation rocks are a typical sand-shale sequence and are fully saturated with water. An example of the array dipole waveforms and the corresponding amplitude spectra are presented in Figure 6. The relative decay of the amplitude spectra with increasing travel distance is evident. This is desired for a reliable attenuation estimate. To ensure there is enough bandwidth for a stable inversion and to avoid the interference of the refracted S-wave at low frequency, we choose a frequency window between 4 and 8 kHz to carry out the inversion. The selected window consists of a total of 30 frequency components. The mud property may vary as the temperature/pressure changes. However, compared with the formation shear attenuation, the mud attenuation is much less depth dependent. Therefore, we set the mud attenuation to

### Table 1. Properties of formation rocks and borehole fluid for synthetic modeling.

<table>
<thead>
<tr>
<th></th>
<th>( DT_p ) (( \mu s/m ))</th>
<th>( DT_s ) (( \mu s/m ))</th>
<th>( Q_p )</th>
<th>( Q_s )</th>
<th>( Q_f )</th>
<th>( \rho ) (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast formation</td>
<td>222.2</td>
<td>355.5</td>
<td>100</td>
<td>30</td>
<td>2539</td>
<td>1000</td>
</tr>
<tr>
<td>Slow formation</td>
<td>555.6</td>
<td>1111</td>
<td>100</td>
<td>30</td>
<td>2192</td>
<td></td>
</tr>
<tr>
<td>Borehole fluid</td>
<td>666.7</td>
<td>N/A</td>
<td>300</td>
<td>N/A</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Frequency-dependent sensitivity coefficients of the flexural wave attenuation.

Figure 4. Percentage error of the inversion results: attenuation of (a) the formation P-wave, (b) the formation S-wave, and (c) the P-wave in the fluid.

Figure 5. Workflow for multifrequency inversion of S-wave attenuation from dipole sonic waveforms.
be constant every 6 m, i.e., every 20 depth points in the inversion procedure. Because the dipole waveforms show a minimum sensitivity to the P-wave, we directly use the monopole P-wave attenuation as a further constraint. In Figure 7, we show the inversion results at four different depth locations. To examine the validity of the results, we model the flexural wave attenuation (using equation 4) based on the inverted S-wave attenuation and compare it with the estimated attenuation. There is an overall good agreement between the two within the chosen frequency range. Results for the entire log are shown in Figure 8. For comparison, we also plotted the P-wave attenuation log computed from the monopole waveforms using a median-filtering method (Frazer et al., 1997).

Figure 8 shows that the dipole shear attenuation (with an average $Q_s$ of 12.5) is overall larger than the monopole compressional attenuation (with an average $Q_p$ of 15.9) across the formations. The high attenuating character of these formations is confirmed by the observation of strong amplitude decay with increasing offsets, as shown in Figure 6. The observation on a $Q_p/Q_s$ ratio slightly larger than one is consistent with the results of Klimentos (1995). The $Q_p/Q_s > 1$ indicates that the formation rocks are fully saturated with water. In addition, the compressional and shear attenuation logs show a negative correlation with the compressional and shear slowness logs (or a positive correlation with the velocity logs). The degree of correlation is stronger for the compressional attenuation than the shear attenuation. A similar observation was made by Best et al. (1994), who found the compressional and shear attenuation correlate with their velocity counterparts for water-saturated sandstones at ultrasonic frequencies.

The inherent coupling between the changes of the attenuation and the velocity are related to the change of the rock properties, such as the lithology. Indeed, we found that the obtained attenuation logs (as well as the velocity logs) exhibit a significant degree of correlation with the gamma-ray log. The sand sections correspond to a higher P- and S-wave attenuation, whereas the attenuation in the shale sections is considerably lower. All shale formations have close to an identical attenuation value with an average of 17 for $Q_s$ and 29 for $Q_p$. The attenuation of the shaly sandstones resides between the attenuation of the sandstones and the shales, which shows a more complicated relation with the gamma-ray value. Information such as the clay content from the corrected gamma-ray could be more useful to characterize the behavior of the attenuation in the shaly sandstones (Klimentos and McCann, 1990; Best et al., 1994). Nonetheless, the correlation between the lithology and formation attenuation clearly demonstrates that the attenuation logs incorporate information about the formation properties. Therefore, the attenuation logs are useful as they can be interpreted with other log data for improved diagnosis of the formation lithology.

Figure 9 shows the comparison between the dipole shear attenuation log and the S-wave attenuation log computed from the monopole waveforms using the median-filtering method (Frazer et al., 1997). The monopole shear attenuation is consistently higher than the shear attenuation estimated from dipole waveforms using our method. This is because the median-filtering method ignores the additional energy loss associated with the geometric spreading of the monopole refracted S-waves. The geometric spreading effect becomes significant if the velocity and the density vary heavily in the chosen section. Therefore, the median-filtering method usually works well on the data acquired from a relatively homogeneous section. In the field-data example, as the sand-shale sequence makes the formation heterogeneous, the monopole shear attenuation produced by the median-filtering method deteriorates. The geometric spreading effect may be removed; however, it requires additional synthetic data modeling and calibration (Sams and Goldberg, 1997).
In this case, the inversion method that we developed for obtaining the shear attenuation from the dipole data shows its advantage in practical applications.

DISCUSSION

With an appropriate modification, the frequency-dependent attenuation analysis can also be applied to other borehole dispersive wave modes, such as Stoneley waves. Decoding the frequency-dependent attenuation and slowness of the Stoneley wave provides a more controlled measure on the formation permeability (Cheng and Cheng, 1996). This serves as one of our future interests. In the current study, the dipole attenuation estimate is only carried out for one component of the dipole data, that is, the waveforms recorded by the inline X-ray transmitter-receiver array. Shear attenuation obtained from full 4C cross-dipole data is potentially useful for formation fracture detection (Hardin et al., 1987). Shear attenuation when applied together with P-wave attenuation is another effective tool for hydrocarbon detection. All of these applications rely on the quality of the sonic waveform data and the robustness of the attenuation extraction method.

The presence of the sonic-logging tool can potentially affect the attenuation estimation. Intuitively, the logging tool reduces the effective diameter of the borehole (Cheng and Toksöz, 1981). As a result, the characteristics of the dispersion and the attenuation curves will change. The logging tool does have an effect, but the effect is different and tool-dependent. The effect of the tool can be added by following the same general approach (Lee et al., 2016). Compared with the tool effects, there also exist other important factors that can influence the sonic log shear attenuation, such as anisotropy and mud invasion (Tang and Cheng, 2004). The above issues are the subjects of our future research. In the current paper, we attempt to focus on analyzing the frequency-dependent dipole attenuation. As a first step to this problem, we pointed out and solved the more impending issue, that is, the contamination of the refracted S-wave to the flexural wave attenuation at low frequency. This lays a practical foundation for accurately determining the shear attenuation from the dipole sonic log data.

CONCLUSION

The flexural wave attenuation (slowness) exhibits frequency dependence and is controlled by the shear attenuation (slowness) at low frequency and the mud attenuation (slowness) at high frequency. In a slow formation where the refracted S-wave is absent, the formation shear attenuation can be directly identified from the dipole attenuation estimate at the low frequencies. In a fast formation, the additional amplitude loss due to the propagation of the refracted S-wave leads to a significant discrepancy between the waveform attenuation estimate and the flexural wave attenuation (or the formation shear attenuation) at low frequencies. This precludes a direct determination of the shear attenuation from the low-frequency part of the waveforms. Consequently, ignoring the additional amplitude loss associated with the refracted S-wave can lead to substantial overestimation of the formation shear attenuation. However, at higher frequencies, the flexural wave propagates at a velocity slower than the formation S-wave velocity and is thus
not contaminated by the refracted S-wave arrival. Based on this property, we proposed a method that uses a multifrequency inversion of the flexural wave attenuation for determining the shear attenuation. The inversion is conducted in a higher frequency part of the dipole waveforms (i.e., greater than 4 kHz for a fast formation). We have demonstrated the accuracy of the proposed method on the synthetic data and the field data.

**ACKNOWLEDGMENTS**

We acknowledge the EDB Petroleum Engineering Professorship for financial support. Y. E. Li is funded by MOE Tier-1 grant R-302-000-165-133.

**DATA AND MATERIALS AVAILABILITY**

Data associated with this research are confidential and cannot be released.

**REFERENCES**


