Least-squares reverse time migration with random space shift (RSS-LSRTM)

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Outline

- Introduction
- Methodology
- Numerical Examples
  - Layered model
  - Marmousi model
- Conclusions
Outline

● Introduction

● Methodology

● Numerical Examples
  ➢ Layered model
  ➢ Marmousi model

● Conclusions
LSRTM: Higher resolution, balanced amplitudes, reduced artifacts

$m_{mig}(x) = L_0^T d$

$L_0$: forward Born modeling operator
d: observed data  T: transpose
m: reflectivity $m_{mig}$: migration image

$f(m) = \frac{1}{2} \| L_0 m - d \|_2^2$

Forward-propagation  Backward-propagation

RTM

Distance(km)

Depth(km)

0 1 2 3 4

0 0.5 1 1.5

Dai et al., 2010; Wong et al., 2011; Yang et al., 2016

Inverse problem

LSRTM

Distance(km)

Distance(km)
Assumption in LSRTM

Accurate migration velocity model

Reality in field data application

Inevitable velocity error in the migration velocity model (Jones et al., 2017)
- Noises in the observed data
- Approximation of the physics in data processing
- Non-uniqueness of the inverse problem for velocity model building

Velocity errors may be larger than 5% (Gray, 2016)
Potential solution:

➢ Model extension for data fitting
➢ Model reduction for geologically meaningful image

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Disadvantage:

➢ Increased memory requirements and computational costs
True velocity model

10% velocity error for migration

Scaling factor: 0.9
ERTM

Homogeneous $v=0.9\; v_t$

Sub-offset gather

$h = -400\text{m}:400\text{m}$

Model extension
Homogeneous $v = 0.9 \, v_t$

Sub-offset gather

$h = -400\text{m}:400\text{m}$

Model extension

LSERTM

SOCIGs at $x=2000\text{m}$

SOCIGs at $z=750\text{m}$

LSERTM image at $h=0$
Model reduction

Stacked LSERTM

Homogeneous
\( v = 0.9 \, v_t \)

Partial offsets
\( h = -50m:50m \)

\[
h_{\text{max}} = \frac{1}{2} \lambda = \frac{1}{2} \frac{\nu}{f}
\]
Blue line:
LSRTM
Inaccurate velocity

Red line:
LSERTM
Inaccurate velocity

Black line:
LSRTM
Accurate velocity
Objective:

➢ Better focused image than conventional LSRTM

➢ Reduced memory requirements and computational costs compared to LSERTM

LSRTM with random space shift (RSS-LSRTM)

Introduce implicit model extension and immediate model reduction at each iteration of the inversion procedure
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**Methodology**

**Extended LSRTM**

\[
m_{\text{mig}}(x,h) = \int dx_s dx_r dt d\tau \frac{\partial^2}{\partial t^2} f_s(\tau) G(x-h,x_s,\tau) G(x+h,x_r,t-\tau) d(x_s,x_r,t)
\]

\[
u(x_s,x_r,t) = \int dh dx d\tau \frac{\partial^2}{\partial t^2} f_s(\tau) G(x-h,x_s,\tau) G(x+h,x_r,t-\tau) m_{\text{mig}}(x,h)
\]

\[
f(m,h) = \frac{1}{2} \|u-d\|^2
\]

Lateral migration resolution \(\frac{\lambda}{2}\)

**Stacked LSERTM**

\[
m_{\text{mig}}(x) = \sum_h m_{\text{mig}}(x,h) \quad h \in \left[-\frac{\lambda}{2}, \frac{\lambda}{2}\right]
\]

Accurate velocity

Inaccurate velocity
Extended imaging condition with stacking

Random-space-shifted LSRTM

\[
m_{\text{mig}}(x) = \frac{1}{2} \frac{y_{\text{max}}}{f} \frac{1}{2} \frac{y_{\text{max}}}{f}
\]

Computational cost saving:

\[
N_h = \frac{y_{\text{max}}}{f} + 1
\]

\[
h_r \in \left[ -\frac{\lambda}{2}, \frac{\lambda}{2} \right]
\]

Denoising: f-x RNA (Liu et al., 2012)
Outline

⚫ Introduction

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⚫ Numerical Examples
  ➢ Layered model
  ➢ Marmousi model

⚫ Conclusions
Layered model

Model size: 401 x 151
Grid interval: 10 m
Peak freq: 30 hz

Sources: 79
Source spacing: 50 m
Receivers: 401
Receiver interval: 10 m
Layered model

RTM

Homogeneous $v=0.9 \, v_t$

LSRTM
Layered model

Stacked ERTM

Homogeneous $v=0.9 \, v_t$

Stacked LSERTM
Layered model

RSS-RTM

Homogeneous $v=0.9 \, v_t$

RSS-LSRTM
Layered model

The computational costs and memory requirements are the same as LSRTM

The image quality is geologically comparable to stacked LSERTM

RSS-RTM Denoising (Liu et al., 2012)

Homogeneous \( v = 0.9 \, v_t \)

RSS-LSRTM Denoising (Liu et al., 2012)
Marmousi model

Model size: 461 x 170
Grid interval: 10 m
Peak freq: 20 hz
Sources: 89
Source spacing: 50 m
Receivers: 461
Receiver interval: 10 m
Marmousi model
Marmousi model

Stacked ERTM

Stacked LSERTM
Marmousi model

RSS-RTM

RSS-LSRTM
Marmousi model

The computational costs and memory requirements are the same as LSRTM

The image quality is geologically comparable to stacked LSERTM

RSS-RTM
Denoising (Liu et al., 2012)

RSS-LSRTM
Denoising (Liu et al., 2012)
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Conclusions

- We have proposed an LSRTM algorithm with random space shifts: RSS-LSRTM

- RSS-LSRTM provides a subsurface image with better continuity and less migration artifacts than conventional LSRTM

- The computational costs and memory requirements of RSS-LSRTM are the same as conventional LSRTM

- The spirit is to introduce implicit model extension and immediate model reduction at each iteration of the inversion procedure
THANKS FOR YOUR ATTENTION!

Email: ceeyaj@nus.edu.sg
Blue line:
LSRTM
Inaccurate velocity

Red line:
RSS-LSRTM
Inaccurate velocity

Black line:
LSERTM
Inaccurate velocity
Red line: RSS-LSRTM Inaccurate velocity
Blue line: LSRTM Inaccurate velocity
Black line: LSERTM Inaccurate velocity
Introduction: synthetic examples of LSRTM

True velocity model

3000 m/s

3150 m/s

RTM accurate migration velocity
Introduction: synthetic examples of LSRTM

**True velocity model**
- 3000 m/s
- 3150 m/s

**LSRTM accurate migration velocity**
- 3000 m/s
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Introduction: synthetic examples of LSRTM

True velocity model

- 3000 m/s
- 3150 m/s

RTM inaccurate migration velocity (10% error)
Introduction: synthetic examples of LSRTM

True velocity model

3000 m/s

3150 m/s

LSRTM
inaccurate migration velocity
(10% error)
Blue line:
LSRTM
Inaccurate velocity

Black line:
LSRTM
Accurate velocity
LSERTM

Homogeneous $v=0.9\ v_t$

Sub-offset gather

$h = -400\text{m}:400\text{m}$
Stacked LSERTM

Homogeneous
$v = 0.9 \, v_t$

Partial offsets
$h = -50m:50m$

$h_{\text{max}} = \frac{1}{2} \lambda = \frac{1}{2} \frac{v}{f}$
Layered model

Distance (m)

Depth (m)

0 1000 2000 3000 4000
0 500 1000 1500

LSRTM
Homogeneous
\( v = 0.9 \, v_t \)
Layered model

Stacked LSERTM

Homogeneous
\( v = 0.9 \, v_t \)

\( h = -400m:400m \)

\( N_h = 80 \)
Layered model

RSS-LSRTM

Homogeneous $v = 0.9 \, v_t$

The computational costs and memory requirements are the same as LSRTM.
Layered model

RSS-LSRTM

Homogeneous
\( v = 0.9 \, v_t \)

Denoising: fx-RNA
(Liu et al., 2012)
Marmousi model

LSRTM

Inhomogeneous

$v = 0.9 \, v_t$
Marmousi model

Stacked LSERTM

Inhomogeneous

\( v = 0.9 \, v_t \)

\( h = -400m:400m \)

\( N_h : 80 \)
Marmousi model

The computational costs and memory requirements are the same as LSRTM

RSS-LSRTM

Inhomogeneous $v=0.9 \, v_t$
Marmousi model

RSS-LSRTM

Inhomogeneous

\( v = 0.9 \, v_t \)

Denoising: f-x RNA

(Liu et al., 2012)
Recorded data

Modeled data LSRTM

Modeled data LSERTM

Modeled data RSS-LSRTM
Methodology

Conventional LSRTM

\[ m_{\text{mig}}(x) = \int dx_s dx_r d\tau \frac{\partial^2}{\partial \tau^2} f_s(\tau) G(x, x_s, \tau) G(x, x_r, t - \tau) d(x_s, x_r, t) \]

\[ u(x_s, x_r, t) = \int dx d\tau \frac{\partial^2}{\partial \tau^2} f_s(\tau) G(x, x_s, \tau) G(x, x_r, t - \tau) m_{\text{mig}}(x) \]

\[ f(m) = \frac{1}{2} \left\| u - d \right\|^2 \]

\[ f_s(t): \text{source function} \quad G(x,t): \text{Greens Functions} \quad x_s: \text{source location} \]

\[ x_r: \text{receiver location} \quad d: \text{observed data} \quad u: \text{modeled data} \]

Extended LSRTM

\[ m_{\text{mig}}(x, h) = \int dx_s dx_r d\tau \frac{\partial^2}{\partial \tau^2} f_s(\tau) G(x - h, x_s, \tau) G(x + h, x_r, t - \tau) d(x_s, x_r, t) \]

\[ u(x_s, x_r, t) = \int dh dx d\tau \frac{\partial^2}{\partial \tau^2} f_s(\tau) G(x - h, x_s, \tau) G(x + h, x_r, t - \tau) m_{\text{mig}}(x, h) \]

\[ f(m(x, h)) = \frac{1}{2} \left\| u - d \right\|^2 \]

\[ f_s(t): \text{source function} \quad G(x,t): \text{Greens Functions} \quad x_s: \text{source location} \]

\[ x_r: \text{receiver location} \quad d: \text{observed data} \quad u: \text{modeled data} \]
**Methodology**

**Extended LSRTM**

\[ m_{\text{mig}}(x,h) = \int dx_s dx_r dt d\tau \frac{\partial^2}{\partial t^2} f_s(\tau) G(x-h, x_s, \tau) G(x+h, x_r, t-\tau) d(x_s, x_r, t) \]

\[ u(x_s, x_r, t) = \int dx d\tau \frac{\partial^2}{\partial t^2} f_s(\tau) G(x-h, x_s, \tau) G(x+h, x_r, t-\tau) m_{\text{mig}}(x) \]

**Random-space-shifted LSRTM**

\[ f(m,h) = \frac{1}{2} \|u-d\|^2 \]

\[ f(m(x)) = -\int dx_s \int dx_r \frac{\sum\sum u \cdot d}{\sqrt{\sum\sum u^2} \sqrt{\sum\sum d^2}} \]

**Stacked LSERTM**

\[ m_{\text{mig}}(x) = \sum_h m_{\text{mig}}(x,h) \]

\[ h \in \left[ -\frac{\lambda}{2}, \frac{\lambda}{2} \right] \]

**Lateral migration resolution**

\[ h_s \in \left[ -\frac{\lambda}{2}, \frac{\lambda}{2} \right] \]

**Denoising: f-x RNA** (Liu et al., 2012)

\[ f \text{-x RNA} \]
**Meth**

**Extend**

\[ m_{mig}(x,h) \]

\[ u(x_s,x_r) \]

\[ \sum_h m_{mig}(x,h) \]

\[ h \in \left[ -\frac{\lambda}{2}, \frac{\lambda}{2} \right] \]

**Stacked LSERTM**

**Stacked LSERTM image**